Trajectories Cont.

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def: given $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$

the acceleration @ time t is:

$$\vec{a}(t) = \langle x(t)'', y(t)'', z(t)'' \rangle = \frac{d\vec{v}(t)}{dt} = \frac{d^2\vec{r}(t)}{dt^2}$$ (vector that depends on t)

(by newton's 2nd law $\vec{a}(t)$ is proportional to force)

ex 4) particle moving in space with $\vec{r}(t) = \langle 3t^2 - 5, \cos(t), \ln(t^2) \rangle$, $t > 0$

find velocity & acceleration @ $t=10$ & θ @ $t=10$

solution: velocity $\rightarrow$ $\vec{v}(t) = \langle 6t, -\sin(t), \frac{1}{t^2} \cdot 2t \rangle$ (chain rule !!) (tangent to position vector)

acceleration $\rightarrow$ $\vec{a}(t) = \langle 6, -\cos(t), \frac{-2}{t^2} \rangle$

@ $t=10$ we have ...

$$\vec{v}(10) = \langle 60, -\sin(10), \tfrac{1}{5} \rangle$$
$$\vec{a}(10) = \langle 6, -\cos(10), \tfrac{-1}{50} \rangle$$

use dot product to get θ:

$$|\vec{v}(10)| = \sqrt{60^2 + (-\sin(10))^2 + \tfrac{1}{5}^2}$$

$$|\vec{a}(10)| = \sqrt{6^2 + (-\cos(10))^2 + (\tfrac{-1}{50})^2}$$

$$\vec{v}(10) \cdot \vec{a}(10) = (60 \cdot 6) + (-\sin(10) \cdot -\cos(10)) + (\tfrac{1}{5} \cdot \tfrac{-1}{50})$$

$$\vec{v}(10) \cdot \vec{a}(10) = |\vec{v}(10)| \cdot |\vec{a}(10)| \cos\theta$$

$$\theta = \arccos\left(\frac{\vec{v}(10) \cdot \vec{a}(10)}{|\vec{v}(10)| \cdot |\vec{a}(10)|} \right)$$

$$\theta \approx 80.3^\circ$$